AP Calc AB Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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 WS Assessment

 Target 19:

Intro to differential equation – separation of variables

**I can:**

* Interpret verbal statements of problems as D.E
* Verify solutions to differential equations

Unit 7: Differential Equations

HW Target 19 Unit 7 Progress Check MCQ A

*A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows?*

Let Q(t) is the amount of salt at time t; Q(0) = 5

Rate of water in Rate of water out

Recall: Concentration = amount salt / volume

Volume: start with 600 gallons and every hour 9 gallons enters and 6 gallons leave. So, if we use t in hours, every hour 3 gallons enters the tank, or at any time t there is 600 + 3tt gallons of water in the tank.

Rate of salt = Rate of water (amount of salt)

$$Q^{'}\left(t\right)=9∙0-6∙\frac{Q(t)}{600+3t}$$

Solve this for Q(t).

An equation which includes both a function and its derivative is known as a **differential equation**

*A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. If there are initially 100 insects in the area will the population survive? If not, when do they die out?*

Let P(t) is the population at time t; P(0) = 100 Birth rate is rP(t). Need to find r

Population rate P’(t) = [rP(t) + 15] – [16 + 7]

Solve this for P(t)

Set up the differential equation for the following

*A jogger runs along a straight track. The jogger’s position is given by the function p(t), where t is measured in minutes since the start of the run. During the first minute of the run, the jogger’s acceleration is proportional to the square root of the time since the start of the run. Write a differential equation that describes this relationship, where k is a positive constant?*

*The weight of a population of yeast is given by a differentiable function, W where W(t) is*

*measured in grams and is measured in hours. The weight of the yeast population*

*increases with respect to at a rate that is directly proportional to the weight. At time*

*hours, the weight of the yeast is 200 grams and is increasing at the rate of 5 grams*

*per hour. Write a differential equation that models this situation?*

The order of a differential equation is the same as that of equation's highest-order derivation.

Choose the solution of the given the differential equation

$\frac{dy}{dx}=2x$ a. 2 b. x2 c. 2x2

$\frac{dy}{dx}=\frac{x}{y}$ a. x2y2 = 4 b. x2 – y2 = 4 c. x2 + y2 = 4

$y^{''}-10y^{'}+8y=0$ a. y = 2sin(3x) b. y = 5ex

For what value of k, if any, is y = e2x + ke-3x be a solution to the differential equation

$4y-y^{''}= 10^{-3x}$ a. 10 b. -2 c. 10/3

For what value of k, if any, is y = ksin(5x) + 2 cos(4x) be solution to the differential equation

$y^{''}+16y= -27sin⁡(5x)$ a. -27 b. -9/5 c. 3

$4x^{2}y^{''}+12xy^{'}+3y=0$ a. x -1.5 b. x-.5 c. $7\sqrt{x}$

**Initial Condition(s)** are a condition, or set of conditions, on the solution that will allow us to determine which solution that we are after. Initial conditions (often abbreviated i.c.’s when we’re feeling lazy…) are of the form,

Find the solution of $4x^{2}y^{''}+12xy^{'}+3y=0, y\left(4\right)=\frac{1}{8} and y^{'}\left(4\right)=-\frac{3}{64}$

 a. x -1.5 b. x-.5 c. $7\sqrt{x}$

Show that $y\left(x\right)=\frac{3}{4}+\frac{c}{x^{2}}$ is the general solution to $2xy^{'}+4y=3$

Now find the solution for the problem $y\left(x\right)=\frac{3}{4}+\frac{c}{x^{2}}. y\left(1\right)=-4$

Finding solutions using separation of variable

Solve the following

$\frac{dy}{dx}=\frac{x}{y}$ $\frac{dy}{dx}=\frac{y}{x}$

$\frac{dy}{dx}=\frac{y-2}{x}$ $\frac{dy}{dx}=6y^{2}x y\left(1\right)=\frac{1}{25}$

$\frac{dy}{dx}=\frac{3x^{2}+4x-4}{2y-4}$ y(1) = 3 $\frac{dy}{dx}=\frac{xy^{3}}{\sqrt{1+x^{2}}} y\left(0\right)=-1$

$\frac{dy}{dx}=e^{-y}\left(2x-4\right) y\left(5\right)=0$

$\frac{dy}{dx}=e^{y-x}\sec(\left(y\right))\left(1+x^{2}\right) y\left(0\right)=0$



 

 Find an equation of the curve passing through the point (0, 7) and whose slope at (x, y) is 4x3y

Solve

$\frac{dy}{dx}=x+y$ Hint: Let u = x + y, find u'

$\frac{dy}{dx}=x-y$

$\frac{dy}{dx}-\frac{y}{x}=1$

**Assessment**

 Solve the following

A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows?

A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. If there are initially 100 insects in the area will the population survive? If not, when do they die out?